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OPTIMIZED COMPOUND CURRENT LEADS

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UDC 536.483

A method is given that provides an acceptable approximation to minimizing the energy consumption on the basis of the finite heat-transfer coefficient and the additional heat sources.

Economy and reliability of the current leads are frequently the decisive factors in the design of a cryogenic magnet system. Here we present some results from theoretical studies on optimized leads that enable one to implement designs providing maximum economy and reliability at the drafting stage.

Detailed studies have been made [1-5] on current leads by means of the recuperation coefficient β, which represents the criterion for nonideal cooling. It is assumed that the recuperation coefficient is known from experiment for leads of constant cross section and that the value is in the range $0.5 \leq \beta < 1$; the heat-transfer coefficient defines the actual cooling process, and this can be used in a method in which β is a specified criterion for minimizing the energy consumption [6]. Then the recuperation coefficient is governed by the specified temperature differences along the normal part of the current lead. The method allows one to use a specified deviation from minimum energy consumption in calculating the geometrical parameters of the lead when this is of variable cross section and the local heat-transfer coefficients are known.

We consider the heat-balance equation for a lead in the stationary one-dimensional approximation [1-5]:

$$\frac{dq}{dT} = c_p m \frac{dT_r}{dT} - \frac{I^2 \rho \lambda}{q}, \tag{1}$$

$$\frac{dx}{dT} = \frac{\lambda S}{q} \tag{2}$$

and the expression for the current dimensions, which is derived by transforming (2) in conjunction with the equation for the heat balance involving the cooling vapor in the steady-state one-dimensional approximation.

We also assume that the thermal conductivity of the cooling vapor is negligible and that the heat-transfer conditions are identical over the entire surface of the lead:

Technical Physics Institute, Academy of Sciences of the Ukrainian SSR, Donetsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 2, pp. 360-365, August, 1979. Original article submitted June 13, 1978.

 0022-0841/79/3702-0992 $07.50 © 1980 Plenum Publishing Corporation

$$PS = \frac{c_p m \frac{dT_r}{dT} q}{\lambda a (T - T_r)}. \tag{3}$$

We assume that the shape of the cross section is given:

$$S = f(P). \tag{4}$$

The boundary conditions are

$$x = 0, \quad T(0), \quad T_r(0) = T_b, \quad x = L, \quad T(L), \tag{5}$$

where T(0) is determined from the condition for heat transfer at the cold end of the lead:

$$q(0) = a_l P_l (T(0) - T_b). \tag{6}$$

At the boundary between the normal and superconducting parts of the lead

$$x = l, \quad T(l) = T_s, \quad T_r(l). \tag{7}$$

The boundary conditions for optimality are

$$q(l) = 0, \quad T(L) - T_r(L) = \delta. \tag{8}$$

If the physical parameters of the normal part follow the Wiedeman – Franz law, then we put dT_r/dT = const in (1) and can use the solution of [2].

In the case of current leads in a cryogenic magnet system

$$m = \frac{q(0) + \Phi}{r}. \tag{9}$$

We substitute (9) into (1) and (3) and convert to the specific heat flux to get

$$\frac{dB}{dT} = c_p \frac{dT_r}{dT} \left(\frac{B(0) + K}{r} \right) - \frac{\rho \lambda}{B}, \tag{10}$$

$$PS = \frac{c_p \frac{dT_r}{dT} (B(0) + K)}{\lambda (T - T_r) r} \frac{I^2}{a}, \tag{11}$$

i.e., the distribution of the specific fluxes by temperature level is independent of the current, while the product of the perimeter by the cross section is proportional to the square of the current and inversely proportional to the heat-transfer coefficient.

If a current lead is made of a thin strip ($P = 2h_1$, $S = h_1 h_2$),

$$h_1 = \frac{I^2}{a} F(T), \tag{12}$$

$$L - l = \int_{T(l)}^{T(0)} \frac{\varphi(T)}{a} dT, \tag{13}$$

i.e., the current width of the lead is directly proportional to the current and inversely proportional to the heat-transfer coefficient, while the length is independent of the current and is inversely related to the heat-transfer coefficient.

As there is no heating in the superconducting part, we assume that (5) and (7) are met by appropriate extension of the heat-transfer areas.

Then the mass flow rate of (9) is determined by the conditions at the boundary between the superconducting and normal parts:

$$m = \frac{q(l) + \Phi}{r + i(l) - i_b} \tag{14}$$

or in terms of the specific mass flow rate

$$M = \frac{B(l) + K}{r + i(l) - i_b}. \tag{15}$$

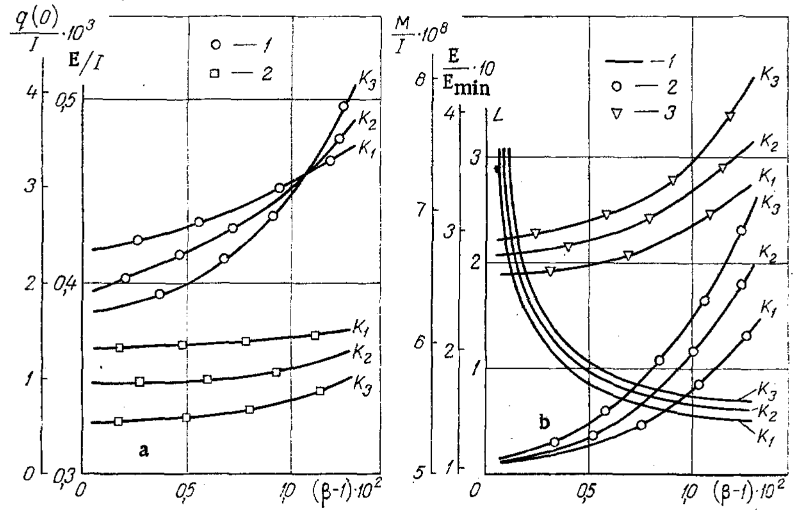

Fig. 1. Dependence on $\beta - 1$ of: a) energy consumption E/I, W/A (1); heat flux at the cold end q(0)/I, W/A (2); b) length L, m (of the normal part of the lead) (1); relative energy consumption E/E_{min} (2); mass flow rate M/I, kg/sec·A (3) for various additional heat fluxes into the liquid helium: $K_1 = 0$; $K_2 = 4 \cdot 10^{-4}$ W/A; $K_3 = 8 \cdot 10^{-4}$ W/A.

The definition of the recuperation coefficient is [5]:

$$\beta = \frac{dT_r}{dT}. \tag{16}$$

We now consider some possible forms of cooling for the normal part.

We assume that

$$\beta = \frac{T_r(L) - T_r(l)}{T(L) - T_s}. \tag{17}$$

This assumption substantially simplifies the analysis without loss of generality, since any smooth function can be approximated by a piecewise-linear one. It follows from (17) that $\beta > 1$ since $T_s > T_r(l) + \delta$ on all occasions.

Figure 1a shows the necessary energy consumption, the specific heat flux, and the specific mass flow rate as functions of $\beta - 1$ for various K as derived by numerical solution by computer of (2), (10), (11), and (15). The material of the normal part is brass [6] and the transition temperature for the superconductor is 8°K.

As $\beta = 1$ corresponds to cooling the current lead with minimum energy consumption, we have to ensure that $T_s - T_r(l) = \delta$.

Maximal energy consumption is involved in implementing the process with $T_r(l) = T_b$ and β_{max}.

The physical meaning of this is that maximum use must be made of the cold content of the cooling vapor over the superconducting part in order to conduct the process with the minimum possible energy consumption while maintaining set temperatures in the lead. In that case, one can provide stable points of transition along the current lead and reduce the dependence on the variable level of the liquid helium by utilizing the vapor with a transverse radiator having an extensive surface, which is installed at the transition point. The calculations on this radiator are performed by standard methods [6].

Figure 1b shows the length of a strip lead ($h_2 = 5 \cdot 10^{-4}$ m) as a function of $\beta - 1$ for various K; the difference in temperature between the current lead and the cooling vapor at the warm end is 0.05°K, while the heat-transfer coefficient is $\alpha = 9 \cdot 10^2 \lambda_r$.

The trend in the curves in the region $\beta - 1 \approx 0$ confirms the obvious fact that implementation of the ideal process is meaningless for a finite value of the heat-transfer coefficient, since then $L \to \infty$.

The relative excess over the minimum consumption is a function of $\beta - 1$, and the values for different K allow one to utilize a given K to draw up an unambiguous relationship between $\beta - 1$ and the deviation from minimum energy consumption. Therefore β, or more conveniently $\beta - 1$, can be used as the criterion for implementing minimum energy consumption.

In the design of a cryogenic magnet, it is sometimes necessary to tap off cooling vapor to cool various additional devices; this case is realized for $K < 0$ in this method.

In order to build a completely gas-cooled current lead it is necessary to provide an elevated value for the specific additional heat flux or else some other source of cooling vapor such that $q(0) \approx 0$.

Figure 1a show that the energy consumption in maintaining the temperature conditions is reduced as the additional source is activated for $\beta \to 1$, i.e., it is better to operate current leads with $K > 0$.

However, Fig. 1b shows that realization of current leads at elevated values of the specific additional heat flux results in an increase in the length of the lead if a given ratio to the minimum energy consumption is to be preserved. In that case, in order to obtain acceptable dimensions it is necessary to increase the heat-transfer coefficient, as (11) shows.

The calculations on the lead are performed in the following stages.

From the given K, which is governed by the design of the cryogenic magnet, and the acceptable deviation, we establish the recuperation coefficient (Fig. 1b).

Equations (2), (11), etc. are solved to determine the geometrical parameters and the temperature distribution along the current lead.

The heat-balance equation is solved for the boundary between the normal and superconducting parts to determine the necessary area of the radiator.

If the length of the lead exceeds the maximum permissible size, then design measures are introduced to increase the heat-transfer coefficient.

The following conclusions are drawn from the above.

1. This method of designing optimized compound current leads allows one to implement a cooling mode with a given deviation from minimum energy consumption.

2. The recuperation coefficient is the criterion for deviation from minimum energy consumption for given specific heat fluxes.

3. The geometrical dimensions can be reduced for large specific heat fluxes or with pure gas cooling provided that the cooling is accelerated and that the lead has an increased ratio of perimeter to cross section (thin strip).

4. In order to reduce the energy consumption it is necessary to accelerate the cooling at the boundary between the superconducting and normal parts by installing additional heat-transfer surfaces.

5. In order to reduce the minimum energy consumption required to maintain the temperature conditions it is necessary to make the maximum possible use of additional heat fluxes in the cooling-vapor source.

NOTATION

x, current coordinate; L, lead length; l, superconducting length of lead; ρ, resistivity of the normal lead; λ, thermal conductivity; c_p, specific heat of coolant vapor; λ_r, thermal conductivity of vapor; α, local heat-transfer coefficient; m, mass flow rate of vapor; $M = m/I$, specific flow rate of vapor; Φ, additional heat inflow; i, vapor enthalpy; i_b, vapor enthalpy at the boiling point; T_r, instantaneous vapor temperature; T_s, superconductor transition temperature; T_b, boiling point; q, heat flux; $B = q/I$, specific heat flux; β, recuperation coefficient; I, current, P, perimeter; S, cross-sectional area; r, heat of vaporization, δ, small quantity; h_1, bandwidth; h_2, band thickness; α_l, heat-transfer coefficient for liquid helium; P_l, wetted perimeter; $K = \Phi/I$, specific additional inflow; T, instantaneous lead temperature.

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ESTIMATING THE RANGE OF APPLICABILITY OF THE HYPERBOLIC THERMAL CONDUCTIVITY EQUATION

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UDC 536.24.02

A generalized thermal conductivity equation is considered. The geometric dimensions of regions in which temperature fields may be described by hyperbolic or parabolic thermal conductivity equations are estimated.

In the last decade wide use has been made of the hyperbolic thermal conductivity equation

$$\tau_r \frac{\partial^2 T(x,\ \tau)}{\partial \tau^2} + \frac{\partial T(x,\ \tau)}{\partial \tau} = a\ \frac{\partial^2 T(x,\ \tau)}{\partial x^2} \tag{1}$$

for description of high-intensity processes. In this equation, proposed in [1], τ_r is the relaxation time; a, thermal diffusivity coefficient; $W = \sqrt{a/\tau_r}$, λ, c, ρ, rate of propagation of heat, the thermal conductivity, the specific heat, and density.

In a layer of material of thickness l we will consider the mathematical model of the thermal conductivity process described by Eq. (1) with initial conditions

$$T(x,\ 0) = \varphi_1(x), \quad \frac{\partial T(x,\ 0)}{\partial \tau} = \varphi_2(x) \tag{2}$$

and boundary conditions

$$\alpha_{i1} \frac{\partial T((i-1)\,l,\ \tau)}{\partial x} + (-1)^i \alpha_{i2} T((i-1)\,l,\ \tau) = \varphi_{2+i}(\tau),\ i = 1,\ 2. \tag{3}$$

The coefficients α_{i1}, α_{i2} take on the values 0 and 1, depending upon the form of the boundary conditions.

Following [2], we construct the solution of the system (1)-(3) in the form

$$T(x,\ \tau) = \sum_{n=1}^{\infty} A_n(\tau) X_n(x) + \Psi(x,\ \tau). \tag{4}$$

An auxiliary, sufficiently smooth function $\Psi(x, \tau)$ which reduces inhomogeneous conditions to homogeneous is constructed in a manner such that

$$\Psi(x,\ 0) = \varphi_1(x), \quad \frac{\partial \Psi(x,\ 0)}{\partial \tau} = \varphi_2(x),$$

$$\alpha_{i1} \frac{\partial \Psi((i-1)\,l,\ \tau)}{\partial x} + (-1)^i \alpha_{i2} \Psi((i-1)\,l,\ \tau) = \varphi_{2+i}(\tau),$$

$$F((i-1)\,l,\ \tau) = 0, \quad i = 1,\ 2,$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 2, pp. 366-370, August, 1979. Original article submitted May 3, 1979.

0022-0841/79/3702-0996$07.50 © 1980 Plenum Publishing Corporation